Computing Hilbert modular forms with nonparitious weight

ANTS XVII – Bernoulli Institute, Rijksuniversiteit Groningen

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Outline

1. Why do we care about "Hilbert modular forms with nonparitious weight"?

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2. Hilbert modular forms in general

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2. Hilbert modular forms in general
3. How do we compute nonparitious forms?

Preamble

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- The icon (🧹) indicates that I am deliberately suppressing a detail for ease of exposition

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Guiding question

What are the automorphic forms associated to interesting families of abelian varieties?

Abelian Varieties

Automorphic forms

Abelian varieties and their automorphic forms

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- Understanding generic abelian varieties seems intractable...

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Can hope to attack (theoretical and computational) modularity questions for atypical abelian varieties

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- Smallest dimension where all four Albert endomorphism types appear

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Why compute nonparitious HMFs?

They provide insight into how nonalgebraic automorphic forms appear in geometry.

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Definition

The weight k is **paritious** if its entries are all congruent mod 2, **nonparitious** otherwise.

q-expansions and Hecke operators on HMFs

- Any Hilbert modular form f has a q-expansion

$$f(z) = \sum_{\nu \in \mathfrak{d}_{>0}} a_\nu(f) q^\nu$$

with coefficients indexed by **elements** of some fractional ideal $\mathfrak{d}$ of F.

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- The Hecke operators $\{T_\mathfrak{n}\}$ (indexed by **ideals** $\mathfrak{n} \subset \mathbb{Z}_F$) commute and act on $M_k(\mathfrak{N}, \chi)$, restrict to $S_k(\mathfrak{N}, \chi)$.

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- **Manipulating q-expansions:** ABBC-Costa-Duque-Rosero-H-Kieffer-M**M**MSST-Voight [ABB$^+$26]

Producing a basis given the Hecke matrices

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This work

- Extend the Donnelly-Voight algorithm to the nonparitious setting.
- Provide an implementation as part of a larger package for computing Hilbert modular forms

How we compute spaces of nonparitious HMFs

Assumptions going forward

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$$a_{\mathfrak{n}}(f) := a_\nu(f) \nu^{\underline{k_0} - k/2} = a_\nu(f) \prod_{i=1}^{n} \nu_i^{k_0 - k_i/2}, \tag{2}$$

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- The $T_{\mathfrak{n}}$ eigenvalue of a normalized () newform f is $a_{\mathfrak{n}}(f)$.

The matrix of $T_{\mathfrak{p}}$

Weight representation

Let ρ be the representation of G, $\Gamma \subseteq G \subseteq \mathrm{GL}_2(F)$ () , given by

$$\chi \otimes \bigotimes_{i=1}^{n} \left(\mathrm{Sym}^{k_i - 2} F^2 \otimes \det^{\frac{k_0 - k_i}{2}} \right) \circ \sigma_i .$$

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Theorem [DV13, AM26]

Pick π totally positive such that $\mathfrak{p} = (\pi)$. $T_{\mathfrak{p}}$ acts on $S_k(\mathfrak{N}, \chi)$ by a block matrix, where each block is a $\mathbb{Q}$-linear combination of matrices $\rho(\varpi)$ for $\varpi \in G$

The matrix of $T_{\mathfrak{p}}$

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Facts

- $T_{\mathfrak{p}}$ is independent of choice of π (because of determinant twist)
- ρ is defined over F when k is paritious (square roots are safe)

Producing q-expansions from Hecke matrices (k paritious)

Corollary

The (matrix for the) operator $T_{\mathfrak{p}}$ on $S_k(\mathfrak{N}, \chi)$ can be defined over F when k is paritious.

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Definition

The Hecke algebra $\mathbb{T}^{\mathrm{new}}$ is the F-algebra generated by the matrices of $T_{\mathfrak{p}}$ acting on $S_k(\mathfrak{N}, \chi)^{\mathrm{new}}$.

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- So there are field extensions K_f/F such that

$$\mathbb{T}^{\mathrm{new}} \cong \prod_f K_f,$$

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  where each f represents a Galois orbit () of newforms
- We can produce the q-expansion of f by considering

$$\begin{tikzcd}
\mathbb{T} \arrow[r, "\sim"] & \prod_f K_f \arrow[r] & K_f \\
T_{\mathfrak{p}} \arrow[rr, mapsto] & & a_{\mathfrak{p}}(f)
\end{tikzcd}$$

Problem in the nonparitious setting

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- So the matrix for $T_{\mathfrak{p}}$ on $S_k(\mathfrak{N}, \chi)$ *cannot* be defined over F
- Doesn't even make sense to define the Hecke algebra over any finite extension!

Resolution – elemental Hecke operators

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Idea: If the determinant factors in ρ are causing problems, just get rid of them!

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- The eigenvalue of T_π on a normalized ( ) newform f is the Fourier coefficient $a_{\pi\delta^{-1}}(f)$, where δ^{-1} is a totally positive generator of $\mathfrak{d}$

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- If k is paritious, $\mathbb{T}^{\mathrm{elem}} \cong \mathbb{T}$
- So $\mathbb{T}^{\mathrm{elem}}$ is the "better" object to work with!

In summary

Problem: If f is nonparitious, the $\{T_\mathfrak{p}\}$ are not all defined over a common number field

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- Prove that the "elemental Hecke algebra" generated by these operators behaves as we hope
- Run the same algorithm as before

Thank you :)
Enjoy your free afternoon!

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