Alternating compositions of weighted differential operators yield the weights' Wronskian

- with which constant?

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THE COLLAPSE · THEOREM 1.1

N! SIGNED TERMS

$$\sum_{\sigma\in S_N} (-1)^\sigma \cdot w_{\sigma(1)}\partial_x{}^p \circ \cdots \circ w_{\sigma(N)}\partial_x{}^p(f)$$

collapses to $\longrightarrow$

ONE UNIVERSAL CONSTANT

$$\mathrm{const}(p) \cdot \mathrm{Wronsk}(w_1,\ldots,w_N) \cdot \partial_x{}^p f$$

N = 2p operators · each of strict order p · p = 1 ⇒ Lie bracket

01 THE ALGORITHM

Prune N! down to the survivors Φ_p

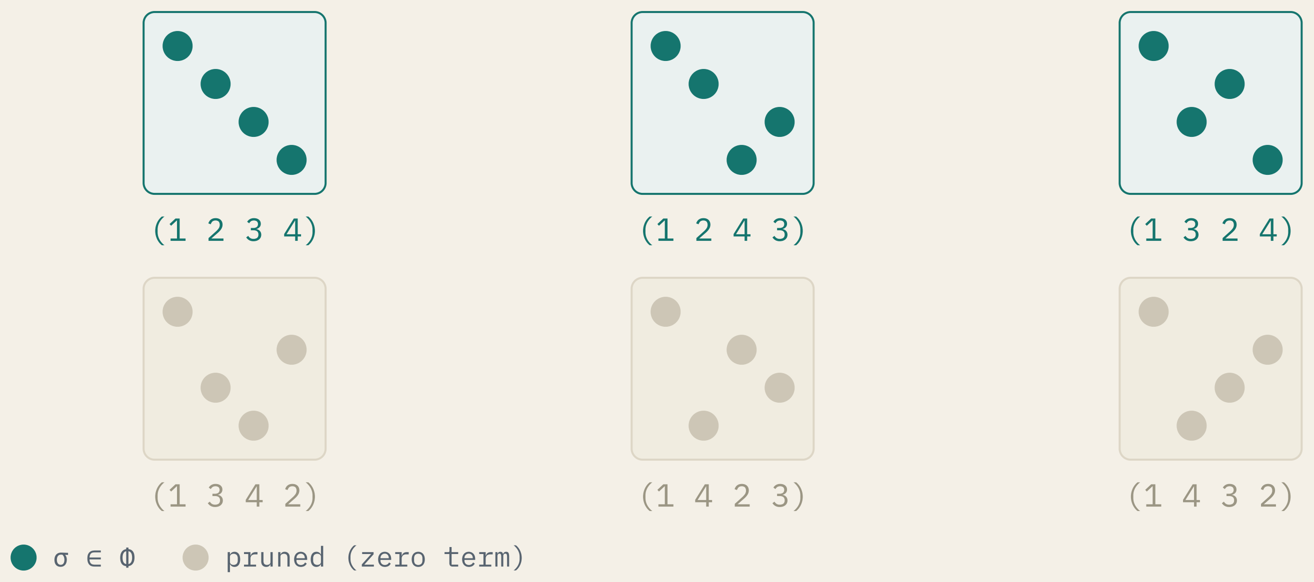
Only permutations whose weights grow *late enough* to survive p differentiations contribute. A backtracking generator constructs exactly those - pruning whole branches the moment the exponent budget goes negative, never enumerating all N! terms.

GENERATOR (VER. 2) · BACKTRACKING

```
fix σ(1) ← 1
fill positions N…2 (right → left) from {2,…,N}:
    try candidate R:
        s ← s + (R - (p+1))  # exponent budget
        if s ≥ 0 : recurse  # branch survives
        else : prune subtree # discard branch
emit σ when pool empty  # σ ∈ Φₚ ⊂ S_N
```

Late-growing. Permutations near the identity present weights in increasing degree and survive; those near the reversed order drive the exponent below zero and vanish.

THE FILTER AT P = 2, N = 4 · 6 CANDIDATES, 3 SURVIVE



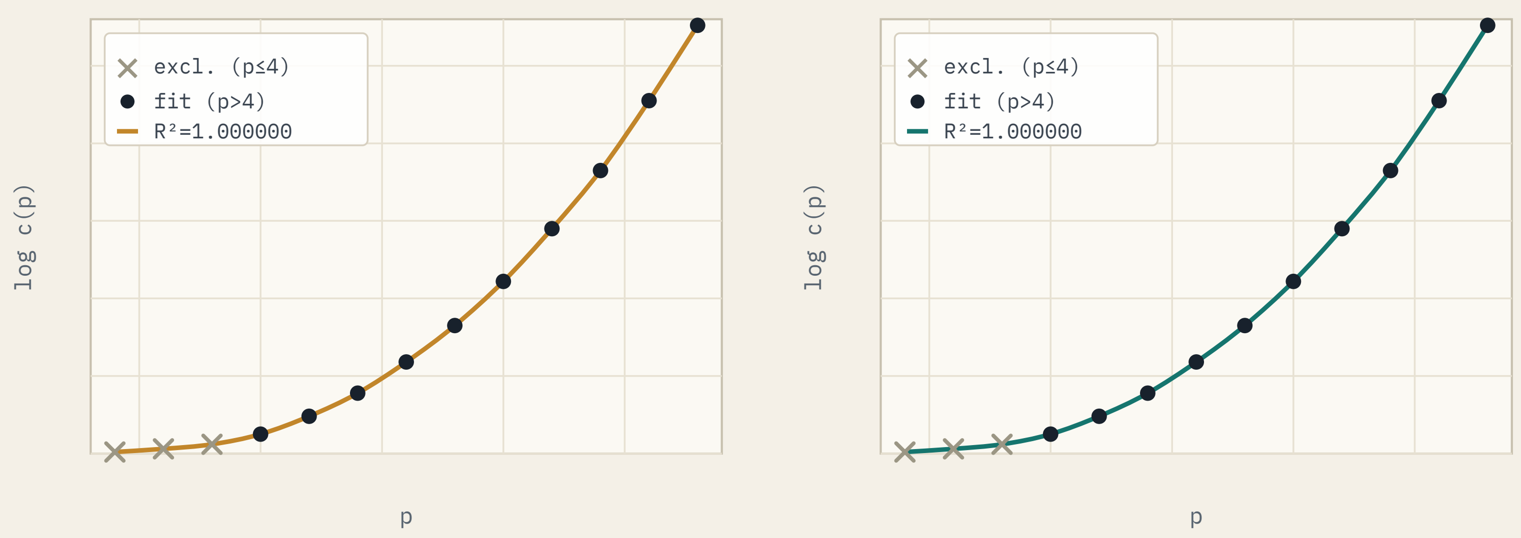
EFFECTIVENESS OF THE FILTER - THE SURVIVING FRACTION COLLAPSES AS P GROWS

p	1	2	3	4	5	6
$\|S_N\|$=N!	2	24	720	40,320	3,628,800	479,001,600
$\|\Phi_p\|$	1	3	35	1,001	53,109	4,605,271
survive	1/2	1/8	~1/20	~1/40	~1/68	~1/104

02 THE SEQUENCE

const(p): a seemingly new integer sequence

EMPIRICAL GROWTH FITS · LOG C(P) VS P · TWO STRONG MODELS



(a) $c\cdot p^2 \log p + d\cdot p^2 + \mathrm{const}$
c = 1.85 d = 2.09 const = 6.18

(b) $c\cdot p \log(p!) + d\cdot\log((2p)!) + \mathrm{const}$
c = 1.79 d = 1.21 const = 3.68

Both models fit the computed values to $R^2 \approx 1$ once the small-p terms ($p \le 4$) are excluded.

P	N	CONST(P)
1	2	1
2	4	2
3	6	90
4	8	586,656
5	10	1,915,103,977,500
6	12	7,886,133,184,567,796,056,800

CONNECTION TO LATE-GROWING PERMUTATIONS

The survivor counts $|\Phi_p|$ = 1, 3, 35, 1001, 53109, 4605271, ... are the even-index terms of OEIS A147681 (late-growing permutations); the sequence **const(p)** itself appears to be new.

03 A P-ADIC LAW

For prime p, $v_p(\mathrm{const}(p)) = p - 1$

v_q(CONST(P)) · PRIMES Q (ROWS) × P (COLUMNS)

q\p	2	3	4	5	6	7	8	9	10	11	12	13	14
2	1	1	5	2	5	4	17	11	16	7	18	13	17
3	·	2	3	1	6	2	2	14	6	5	11	8	3
5	·	1	·	4	2	·	·	·	8	6	5	2	2
7	·	·	1	1	·	6	5	2	·	·	·	·	13
11	·	·	·	·	1	·	·	2	·	10	8	6	4
13	·	·	·	·	·	1	1	1	·	1	·	12	11
17	·	·	·	·	·	·	·	1	·	·	·	1	·
19	·	·	·	·	·	·	·	·	1	1	·	·	·

low — high valuation · ☐ v(const(p)) for prime p = p − 1

CONJECTURE 1

$v_p(\mathrm{const}(p)) = p - 1$

for every prime p - the prime p divides its own constant to the highest order, exactly p-1.

THEOREM 7.6 · PROVED

$v_p(\mathrm{const}(p)) \ge p - 1$

via Legendre's formula on the falling factorials and the ultrametric inequality - half the conjecture, unconditionally.

Writing $\mathrm{const}(p) = p^{p-1}\cdot S_p$, what remains is a single divisibility: $p \nmid S_p$.

Computed on the Hábrók HPC cluster · OEIS A147681

refs: A. V. Kiselev, arXiv:math/0410185 (2004) · A. V. Kiselev, doi:10.1088/1742-6596/3152/1/012044 (2025)