Finding large smooth twins

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Consecutive integers

$$\vdots \qquad \vdots \qquad \vdots$$

$$22529735146513345759959448571 = 2137 \cdot 10542693096169090201197683$$
$$22529735146513345759959448572 = 2^2 \cdot 3 \cdot 907 \cdot 39239 \cdot 70177 \cdot 751717782012361$$
$$22529735146513345759959448573 = 97 \cdot 66533 \cdot 2653633 \cdot 1315547053747481$$
$$22529735146513345759959448574 = 2 \cdot 8423 \cdot 296591 \cdot 4132577 \cdot 1091139724967$$
$$22529735146513345759959448575 = 3^2 \cdot 5^2 \cdot 7^5 \cdot 11^3 \cdot 59 \cdot 71^2 \cdot 101 \cdot 127 \cdot 173^2 \cdot 197 \cdot 199$$
$$22529735146513345759959448576 = 2^{10} \cdot 13 \cdot 17^2 \cdot 23 \cdot 37^2 \cdot 41 \cdot 47 \cdot 61 \cdot 79 \cdot 107^2 \cdot 113^2 \cdot 137$$
$$22529735146513345759959448577 = 4515076051 \cdot 4989890511705435227$$
$$22529735146513345759959448578 = 2 \cdot 3 \cdot 29 \cdot 10273 \cdot 12604033531997919868039$$
$$22529735146513345759959448579 = 73 \cdot 308626508856347202191225323$$
$$22529735146513345759959448580 = 2^2 \cdot 5 \cdot 499 \cdot 599623 \cdot 376484639787767277$$

$$\vdots \qquad \vdots \qquad \vdots$$

"smooth twin"

Can we find large ($\geq$ 240-bit) smooth twins? Such twins should exist with smoothness bounds $B > 1200$ but it is computationally infeasible to find them

Overview of our approach

For $\ell = 38295031104$, all of these are 43000-smooth:

$$\ell \quad \ell+1 \quad \ell+4 \quad \ell+6 \quad \ell+9 \quad \ell+10 \quad \ell+13 \quad \ell+15 \quad \ell+18 \quad \ell+19 \quad \ell^2+19\ell-12$$

$$\left(\frac{(\ell+1)(\ell+4)(\ell+9)(\ell+10)(\ell+15)(\ell+18)(\ell^2+19\ell-12)}{1166400}, \left(\frac{\ell(\ell+6)(\ell+13)(\ell+19)}{1080}\right)^2\right)$$

This is a 262-bit smooth twin since the following polynomials differ by $C = 1166400$

$$f(x) = (x+1)(x+4)(x+9)(x+10)(x+15)(x+18)(x^2+19x-12), \text{ and}$$
$$g(x) = x^2(x+6)^2(x+13)^2(x+19)^2$$

Thanks for listening and enjoy ANTS!