Reconstructing Genus 4 Curves From Their Theta Constants

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Results on genus 4 curves

- *Equations of genus 4 curves from their theta constants*
  - **Input**: Theta constants $\vartheta_C\left[\begin{smallmatrix} a \\ b \end{smallmatrix}\right]$ of a general genus 4 curve C defined over a field k, considered as a point in $\mathbb{P}^{135}_k$.
  - **Output**: A quadric Q and a cubic Γ in $\mathbb{P}^3_k$ such that $C \cong Q \cap \Gamma$
- *Invariants of genus 4 curves*
  - **Input**: A non-hyperelliptic genus 4 curve C over an algebraically closed field K
  - **Output**: Invariants that determine the K-isomorphism class of C
- *Covariant reconstruction of forms from their invariants*
  - **Input**: Invariants of a non-hyperelliptic genus 4 curve over an algebraically closed field K
  - **Output**: A quadric Q and a cubic Γ in $\mathbb{P}^3_K$ such that the genus 4 curve $C = Q \cap \Gamma$ has said invariants

Applications

- Modular Jacobians
- Gluing $2 + 2 = 4$
- CM curves

Example

Let K be the CM field with label 8.0.2147483648.1, given by $x^8 + 8x^6 + 20x^4 + 16x^2 + 2$. It has two CM types, one of which corresponds to a Jacobian of a genus 4 curve. We compute that this is the hyperelliptic curve

$$y^2 = x^9 - 2x^8 - 8x^7 + 16x^6 + 20x^5 - 40x^4 - 16x^3 + 32x^2 + 2x - 4$$

which has CM by $\mathcal{O}_K$.