Some progress On Chinburg's Conj.

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Joint work with Bertin, Hokken, & Ringeling

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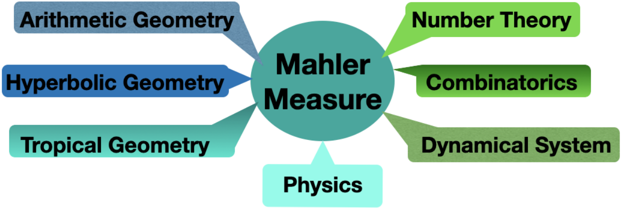

Definition (Mahler, '60)

$$m(P) := \frac{1}{(2\pi i)^n} \int_{\mathbb{T}^n} \log\left|P(z_1, \ldots, z_n)\right| \frac{dz_1}{z_1} \wedge \cdots \wedge \frac{dz_n}{z_n}.$$

$$m(1 + x + y) = L'(\chi_{-3}, -1)$$

(Chinburg's conjecture , '84)

$\forall$ odd quadratic character χ_{-f}, $\exists$ $P_f \in \mathbb{Q}(x, y) \setminus \{0\}$:

$$m(P_f) = r_f L'(\chi_{-f}, -1), \; r_f \in \mathbb{Q}.$$

f	3	4	7	8	11	15	19	20	23
f	24	35	39	40	55	84	120	303	755

Table: Ray'87, Boyd & Rodriguez-Villegas '02, Liu & Hourong '19.

f	3	4	7	8	11	15	19	20	23
f	24	35	39	40	55	84	120	303	755
31	43	51	59	68	111	132	219	228	260

Table: Bertin & M.'23, Hokken & Ringeling & M. '24.

(Generalized Chinburg's conjecture)

$\forall$ odd primitive character χ, $\exists\ P_\chi \in \mathbb{Q}(x,y)\setminus\{0\}$:

$$m(P_\chi) = r_\chi\left(L'(\chi,-1) + L'(\bar{\chi},-1)\right),\ r_\chi \in \mathbb{Q}.$$

f	5	7	9