Pseudorepresentations Not Arising from Genuine Representations

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Pseudorepresentations

Pseudorepresentations of a group G over a commutative ring A are A-valued functions on G that behave like characters of finite-dimensional representations.

Example (dimension $= 2$):

- A pseudorepresentation is a pair of functions (T, D) where T behaves like the trace function and D behaves like the determinant.
- A genuine representation $\rho\colon G \to \mathrm{GL}_2(A)$ has the associated pseudorepresentation $(\mathrm{tr}(\rho), \det(\rho))$.

Embedding Problem

Given a pseudorepresentation (T, D), does there exist a representation ρ whose associated pseudorepresentation coincides with (T, D)?

Embedding Problem

Idea: Fix the residual representation, and then compare the corresponding universal deformation rings via explicit computation.

Previous results: the answer to the embedding problem is yes, if either of the following conditions holds:

- A is an algebraically closed field;
- the residual pseudorepresentation $(\bar{T}, \bar{D})$ is multiplicity-free.

Counterexample

For the extraspecial group $G = 2_{+}^{1+4}$, the deformation rings are not isomorphic to each other, and the answer to the embedding problem is no.