On Adaptive Attacks against Jao-Urbanik's Isogeny-Based Protocol

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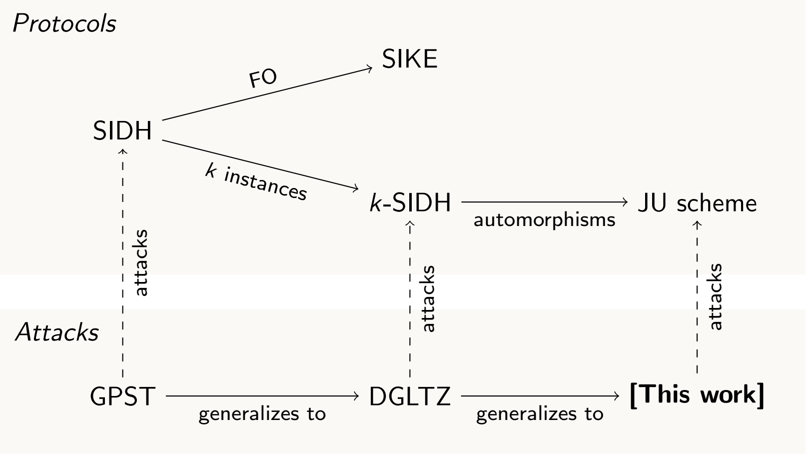
Introduction



Our results

- We exploit the additional structure between curves in the JU scheme to achieve a nearly cubic speed-up when compared to the DGLTZ attack
- Our attack does NOT break the JU scheme for the proposed parameters...
- ...but it shows that at the same security level the JU scheme requires almost twice the computations of k-SIDH to reduce the public-key size by 20%

Additional Information

SIDH [4]

SIDH is a key-exchange protocol over supersingular elliptic curves defined over $\mathbb{F}_{p^2}$, where $p = 2^{e_A}3^{e_B}f \pm 1$.

$$\begin{tikzcd}
 & E_A \arrow[dr, "\phi'_B"] \arrow[d, dashed, shift left, "{E_A,\ \phi_A(P_B),\ \phi_A(Q_B)}"] & \\
E_0 \arrow[ur, "\phi_A"] \arrow[r, phantom] \arrow[dr, "\phi_B"'] & & E_{AB} \\
 & E_B \arrow[ur, "\phi'_A"'] \arrow[u, dashed, shift left, "{E_B,\ \phi_B(P_A),\ \phi_B(Q_A)}"] &
\end{tikzcd}$$

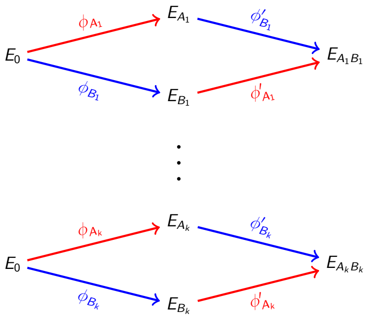
$\langle P_A, Q_A\rangle = E_0[2^{e_A}]$ and $\ker\phi_A = \langle P_A + [\alpha]Q_A\rangle$,
$\langle P_B, Q_B\rangle = E_0[3^{e_B}]$ and $\ker\phi_B = \langle P_B + [\beta]Q_B\rangle$.

GPST attack [3]

- Static secret keys in SIDH can be recovered by a dishonest participant Bob with the adaptive GPST attack
- An attacker uses the key exchange as an oracle to retrieve the static key α of Alice iteratively
- The oracle: returns true if $E_B/\langle R + [\alpha]S \rangle = E_{AB}$, where R, S are the torsion points sent by the attacker Bob
- Sending malicious torsion points R, S the dishonest participant Bob retrieves one bit of α per oracle query
- Countermeasure: Fujisaki-Okamoto or similar transform (as in SIKE)

k-SIDH [1]

k-SIDH avoids attacks such as GPST by performing k^2 instances of SIDH during a single execution of the static-static key exchange protocol.



Using each combination E_{A_i}, E_{B_j} for $i, j = 1, \ldots, k$ of the two parties' k different public curves yields shared secret $\mathrm{Hash}(j(E_{A_1B_1}), j(E_{A_1B_2}), \ldots, j(E_{A_kB_k}))$.

The DGLTZ-attack on k-SIDH [2]

- The attacker queries with the same curve and same extra points for each SIDH instance
- New oracle: returns true if an attacker guesses all the common computed curves correctly
- First step: query with $(E_B, P, [1 + 2^{n-1}]Q)$, one has to query $6 \cdot 7^{k-1}$ times to get the first bit
- With this approach, even for $k = 2$, one needs an exponential number of queries
- DGLTZ solves the issue by computing the intermediate curves and additional points on those curves
- Computing these additional points requires 24^k queries

The Jao-Urbanik protocol I [5]

The protocol improves on k-SIDH by using automorphisms to obtain three instances for each key.

- Starting curve: E_0, $j(E_0) = 0$, with non-trivial automorphism η of order six
- For any subgroup $B \subset E_0$, $E_0/B \cong E_0/\eta(B) \cong E_0/\eta^2(B)$
- Fix bases: $\{P_A, Q_A = \eta(P_A)\}$ of $E_0[2^{e_A}]$, $\{P_B, Q_B = \eta(P_B)\}$ of $E_0[3^{e_B}]$

$$\begin{tikzcd} E_0 \arrow[r, "\phi_A"'] \arrow[d, "\phi_B"'] & E_A \arrow[d, "\phi'_B"] \\ E_B \arrow[r, "\phi'_A"] & E_{A,B} \end{tikzcd}$$

$$\begin{tikzcd} E_0 \arrow[r, "\phi_A"'] \arrow[d, "\phi_{\eta(B)}"'] & E_A \arrow[d, "\phi'_{\eta(B)}"] \\ E_{\eta(B)} \arrow[r, "\phi'_A"] & E_{A,\eta(B)} \end{tikzcd}$$

$$\begin{tikzcd} E_0 \arrow[r, "\phi_A"'] \arrow[d, "\phi_{\eta^2(B)}"'] & E_A \arrow[d, "\phi'_{\eta^2(B)}"] \\ E_{\eta^2(B)} \arrow[r, "\phi'_A"] & E_{A,\eta^2(B)} \end{tikzcd}$$

The Jao-Urbanik protocol II

- Alice and Bob perform SIDH-instance with public keys $(E_A, \phi_A(P_B), \phi_A(Q_B))$ and $(E_B, \phi_B(P_A), \phi_B(Q_A))$

- Alice and Bob obtain as shared secret information (j-invariants of)

$$\left.\begin{array}{l} \blacktriangleright\ E_{A,B} \end{array}\right\} \text{as in standard SIDH}$$

$$\left.\begin{array}{l} \blacktriangleright\ E_{A,\eta(B)} \\ \blacktriangleright\ E_{A,\eta^2(B)} \end{array}\right\} \text{using } \eta \text{ during computation}$$

e.g. Bob uses his secret key β to compute

$E_{A,\eta(B)} = E_A/\langle -\phi_B(P_A) + [\beta+1]\phi_B(\eta(P_A))\rangle$ and

$E_{A,\eta^2(B)} = E_A/\langle -[\beta+1]\phi_B(P_A) + [\beta]\phi_B(\eta(P_A))\rangle$

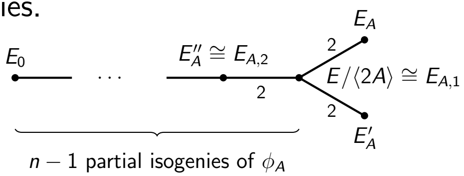
Applying DGLTZ to Jao-Urbanik's protocol

- DGLTZ treats each curve separately
- Secret kernel generators occurring in Jao-Urbanik protocol are not of the required form to straightforwardly apply DGLTZ
- If issues with kernel generators can be overcome, attacking the Jao-Urbanik protocol with k keys and $3k^2$ SIDH-instances would require $\mathcal{O}(24^{3k})$ queries

$\implies$ This work uses relationships between curves and kernel generators to reduce number of queries.

Our attack - First bit recovery

- Goal: get least significant bit α_0 of Alice's secret key α, i.e. determine first curve on isogeny path $E_A \to E_0$.
- Query with $(E_B, [1 + 2^{n-1}]P_B, Q_B)$, so Alice computes all three 2-neighboring curves of $E/\langle 2A \rangle$.
- Underlying relationship between kernel generators of corresponding curves helps to match up triples of candidate curves instead of exhaustively searching over all possibilities.



Our attack - Pullbacks

- Main idea: Let A be a secret kernel, let $E_{A,i}, E'_{A,i}, E''_{A,i}$ be the ith curves on the three corresponding paths.
  Then for all i, the curves $E_{A,i}, E'_{A,i}, E''_{A,i}$ are isomorphic
- Instead of using the DGLTZ attack directly, we compute a pullback candidate for each curve and shift them with the corresponding isomorphisms
- We query the oracle with these related points which saves a lot of time and exploits the extra structure of the scheme

Results

	# SIDH instances	# keys per party	Attack cost
Jao-Urbanik with k keys	$3k^2$	k	$\mathcal{O}(\ell^{5k})$
k-SIDH with $\frac{5}{4}k$ keys	$1.56k^2$	$\frac{5}{4}k$	$\mathcal{O}(\ell^{5k})$

At the same security level, the JU scheme requires almost 2x computations to reduce the public key size by 20%.

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[2] Dobson, S., Galbraith, S.D., LeGrow, J., Ti, Y.B., Zobernig, L.: An adaptive attack on 2-SIDH (2019), http://eprint.iacr.org/2019/890

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[5] Urbanik, D., Jao, D.: New techniques for SIDH-based NIKE (accepted at MathCrypt 2018, to appear in J. Math. Cryptol.; personal communication)