An equivalent problem to the Collatz conjecture

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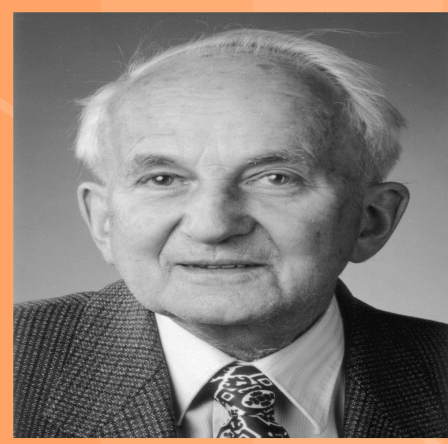
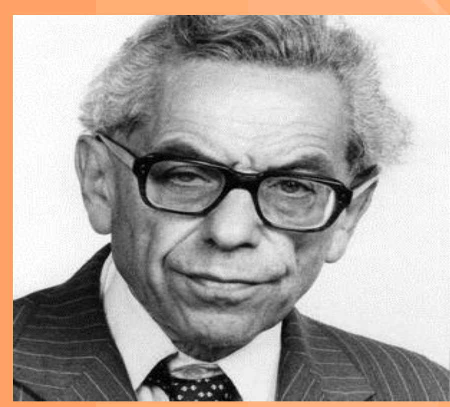
Abstract

In this poster, we give two recurrence sequences of integers, defined by recurrence relations of the form $V_{n+1} = f(V_n; T_n)$ and $T_{n+1} = g(V_n; T_n)$, which are related to the Collatz conjecture.
We also try to understand why the Collatz conjecture is a difficult problem.

Introduction





In 1937, Lothar Collatz posed his famous conjecture saying that for every positive integer U_0, the sequence $(U_n)_{n\in N}$ defined by:

$$U_{n+1} = \begin{cases} \frac{U_n}{2} & \text{if } U_n \text{ is even} \\ 3U_n + 1 & \text{if } U_n \text{ is odd} \end{cases}$$

will always reach the value 1.

Because of the difficulty of resolving Collatz's conjecture, Paul Erdős commented that:

«Mathematics is not yet ready for such problems»

Very recently (2017), Islem Ghaffor found that Collatz’s conjecture is equivalent to the following problem.

Problem

Let V_0 and T_0 be positive integers such that $T_0 \geq \lfloor \log_2 V_0 \rfloor + 1$ and let $(V_n)_{n\in N}$ and $(T_n)_{n\in N}$ be the sequences of integers defined by:

$$V_{n+1} = {V_n}^2 + V_n - (2V_n + 1)\left\lfloor \frac{V_n}{2} \right\rfloor + \left\|2^{T_n} - 3V_n - 1\right\| \left(V_n - 2\left\lfloor \frac{V_n}{2} \right\rfloor\right) 2^{T_n - 1}$$

$$T_{n+1} = T_n - 1 + \left(-\frac{1}{2}\left\|2^{T_n} - 3V_n - 1\right\| + \frac{3}{2}\right)\left(V_n - 2\left\lfloor \frac{V_n}{2} \right\rfloor\right)$$

where $\|x\|$ $(x \in R)$ denotes the quantity: $\|x\| = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x \leq 0 \end{cases}$

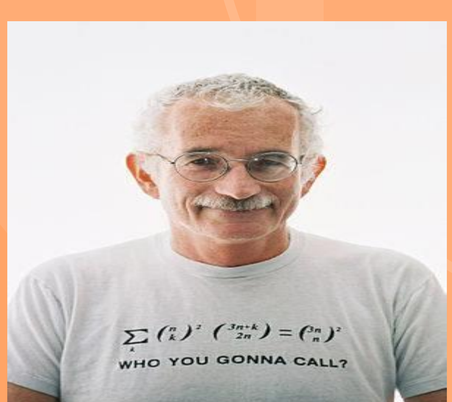
Then we have $\lim_{n\to+\infty} V_n = 0$ for all the values $(V_0; T_0)$.

Remark: It is obvious that if $\lim_{n\to+\infty} V_n = 0$ then $\lim_{n\to+\infty} T_n = -\infty$.

Results

* If we can prove that $\lim_{n\to+\infty} V_n = 0$ for all the values $(V_0; T_0)$ that means Collatz's conjecture is proved.
* The Collatz conjecture is a difficult problem because it is very complicated to calculate $\lim_{n\to+\infty} V_n$.

Acknowledgements



I am thankful to Pr. Doron Zeilberger (Rutgers University, USA) for saying about my equivalent problem that is a nice work.

References

[1] J.C. Lagarias. *The Ultimate Challenge: the $3x + 1$ Problem*, American Mathematical Society, 2010.

Challenge

I am an undergraduate student and I discovered this equivalent problem when I was in high school.
If someone can find out how I created these two recurrence sequences, the prize is $150.

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