Computing the 2-distribution of points on Hermitian surfaces

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Résumé:

A short description is first given of the fascinating use of the Hermitian curve $C(2) : {x_0}^3+{x_1}^3+{x_2}^3 = 0$ of $PG(2, 2^2)$ by the russian mathematician V. Goppa (1981, 1983) in the construction of linear error-correcting codes. Subsequently the work of Goppa inspired many authors.
In 1985, I. Charkravarti suggested a generalization of Goppa construction based on his early work with R. C. Bose on Hermitian varieties by embedding the non-singular Hermitian surface $X(2)$: ${x_0}^3 + {x_1}^3 + {x_2}^3 + {x_3}^3 = 0$ of $PG(3, 2^2)$ in a $PG(9, 2^2)$ via the linear system of quadrics.
In 1986, R. Tobias and P. Spurr by a complete computer search (computer programs) compute the 2-distribution of points on this Hermitian surface $X(2)$. We define by h-distribution of points of a projective variety $\mathcal{V}$ to be the decreasing sequence of the number of points in the section of $\mathcal{V}$ by all hypersurfaces of degree h, associated with their multiplicities.
In 1993, A. B. Sørensen showed that computer program is not necessary in order to find the first family of points in the Hermitian surface $X(2)$ of $PG(3, 2^2)$, and generalized his result on the first family of points in the Hermitian surface $X(t) : {x_0}^{t+1} + {x_1}^{t+1} + {x_2}^{t+1} + {x_3}^{t+1} = 0$ of $PG(3, t^2)$ by the following conjecture:

$$\#X_{Z(f)}(\mathbb{F}_q) \leq h(t^3 + t^2 - t) + t + 1.$$

where f is a homogenous form degree h, and $\#X_{Z(f)}(\mathbb{F}_q)$ the number of points in the section of X by the surface defined by f.
In this poster we will resolve the conjecture of Sørensen for quadric. We will compute the 2-distribution of points on the Hermitian surface $X(t)$. The starting point for the resolution of the above problems is J. W. P. Hirschfeld classification of quadrics. The results of Hirschfeld on the geometry of Hermitian surfaces and quadric surfaces, as well as modifications of methods due to J. P. Serre are used to evaluate the number of points on hypersurfaces, to study $X_Z(f)$ and to show the conjecture for $h = 2$.
A more subtle treatment based on the study on divisors on a surface, and the using of the result on the theorem of Ax on the Zeroes of polynomials over finite fields allow us to appreciate the 2-distribution of points on the surface $X(t)$.

References

[1] Y. Aubry and M. Perret, On the characteristic polynomials of the Frobenuis endomorphism for projective curves over finite fields, Finite Fields and Theirs Applications 10, (2004), 412-431.

[2] I. M. Chakravarti, The generalized Goppa codes and related discrete designs from hermitian surfaces in $PG(3, s^2)$.

Lecture Notes in computer Sci 311. (1986), 116-124.
[3] F. A. B. Edoukou, Codes defined by forms of degree 2 on hermitian surface and Sørensen conjecture, Finite Fields and Their Applications, Volume 13, Issue 3, (2007), 616-627.
[4] F. A. B. Edoukou, Error-Correcting codes constructed from algebraic varieties. Ph. D. Thesis, Université de la Méditerranée, Marseille, France, 2007.
[5] V. Goppa, Algebraico-Geometric codes, V. D. Goppa, Math USSR Izvestiya Vol. 21 (1983) No. 1 75-91.
[6] R. Hartshorne, Algebraic geometry, Graduate texts in mathematics 52, Springer-Verlag, 1977.
[7] J. W. P. Hirschfeld, Finite projective spaces of three dimensions, Clarendon press. Oxford 1985
[8] P. P. Spurr, Linear codes over $GF(4)$. Master's Thesis, University of North Carolina at Chapell Hill, USA, 1986.
[9] I. R. Shafarevich, Basic algebraic geometry 1, Springer-Verlag, 1994.
[10] A. B. Sørensen, Rational points on hypersurfaces, Reed-Muller codes and algebraic-geometric codes. Ph. D. Thesis, Aarhus, Denmark, 1991.