GENUS 2 CURVES WITH SPLIT JACOBIANS

KEVIN DOERKSEN (SIMON FRASER UNIVERSITY)

Poster Abstract

Split Jacobians are special. For genus 2 curves, they can be recognized from the fact that C is a degree n cover of an elliptic curve for some integer n. One can classify split Jacobians of genus 2 curves by these n. If $\psi : C \longrightarrow E$ is a degree n cover then we say $\mathrm{Jac}(C)$ is (n, n) split.

To classify split Jacobians, one can look at all the possible configurations of the ramification points in the covering map $\psi : C \longrightarrow E$ (see for example Kuhn [3] and Shaska [4]). The degree $n = 3$ case was solved by Shaska [6] and studied further in [5]. The degree $n = 5$ case has also been solved in a preprint by Magaard, Shaska, and Völklein. The description of all odd cases greater than $n = 5$ and all even cases greater than $n = 2$ remain open questions. In this poster, we will explain techniques to classify (n, n)-split Jacobians and outline our progress on the degree 4 case.

We begin our poster by showing the degree n cover $\psi : C \longrightarrow E$ of the genus 2 curve C onto the elliptic curve E induces a degree n cover $\phi : \mathbb{P}^1 \longrightarrow \mathbb{P}^1$ such that the following diagram commutes:

$$\begin{array}{ccc} C & \xrightarrow{\ \psi\ } & E \\ {\scriptstyle \pi_C}\big\downarrow & & \big\downarrow{\scriptstyle \pi_E} \\ \mathbb{P}^1 & \xrightarrow{\ \phi\ } & \mathbb{P}^1 \end{array}$$

here, π_C is the natural degree 2 cover of C onto $\mathbb{P}^1$ and π_E is the degree 2 projection of E onto $\mathbb{P}^1$ induced by π_C. Gerhard Frey and Ernst Kani give a complete description of this induced cover ϕ of the projective lines in their 1988 paper [2]. Using this commutative diagram, it is possible to show that if n is odd, then there is only a handful of possible configurations of the ramification points. It was by studying these few possibilities that the $(3, 3)$ and $(5, 5)$ split Jacobians were characterized.

The next section of the poster deals with the case where $n = 4$. In general, the even cases have fewer restrictions on the distribution of the ramification points, and are therefore more difficult to characterize. In order to get around this hurdle, we build up the degree 4 case by first looking at the degree 2 case.

The ability to precisely describe and construct (n, n)-split Jacobians has important computational applications. It allows the construction of abelian varieties with special isogenies and allows new explicit visibility constructions (N. Bruin [1]).

References

1. N. Bruin and E. V. Flynn, *Exhibiting SHA[2] on hyperelliptic Jacobians*, J. Number Theory **118** (2006), no. 2, 266–291. MR MR2225283 (2006m:11091)

This research is performed under the guidance of Professor Nils Bruin (SFU).

2. Gerhard Frey and Ernst Kani, *Curves of genus* 2 *covering elliptic curves and an arithmetical application*, Arithmetic algebraic geometry (Texel, 1989), Progr. Math., vol. 89, Birkhäuser Boston, Boston, MA, 1991, pp. 153–176. MR MR1085258 (91k:14014)
3. Robert M. Kuhn, *Curves of genus* 2 *with split Jacobian*, Trans. Amer. Math. Soc. **307** (1988), no. 1, 41–49. MR MR936803 (89f:14027)
4. T. Shaska, *Curves of genus 2 with* (N, N) *decomposable Jacobians*, J. Symbolic Comput. **31** (2001), no. 5, 603–617. MR MR1828706 (2002m:14023)
5. ———, *Genus 2 curves with* $(3, 3)$*-split Jacobian and large automorphism group*, Algorithmic number theory (Sydney, 2002), Lecture Notes in Comput. Sci., vol. 2369, Springer, Berlin, 2002, pp. 205–218. MR MR2041085 (2005e:14048)
6. ———, *Genus 2 fields with degree 3 elliptic subfields*, Forum Math. **16** (2004), no. 2, 263–280. MR MR2039100 (2004m:11097)

Simon Fraser University