Constructing Nonhyperelliptic Algebraic Function Fields of Genus 3 and 4

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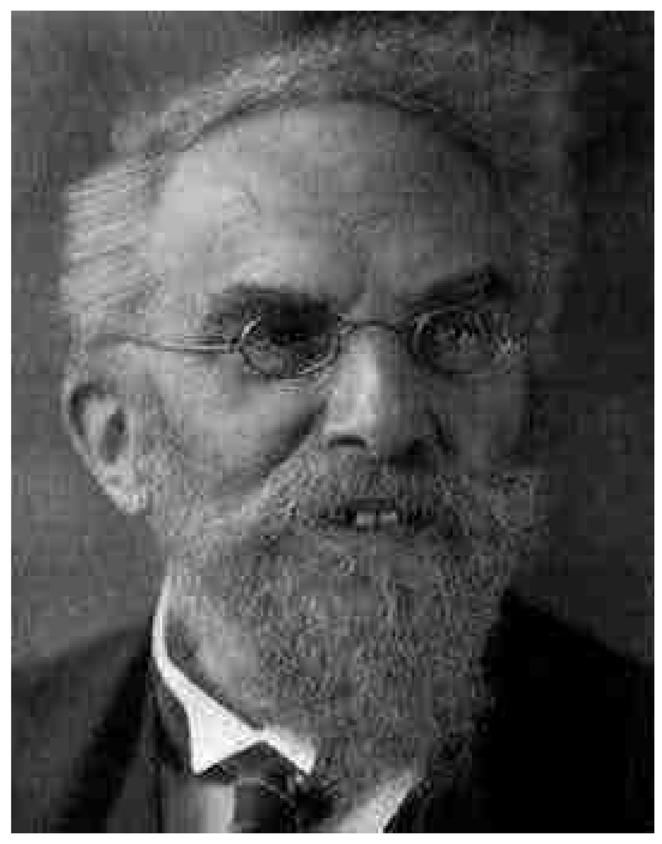
The aim Constructing and classifying nonhyperelliptic algebraic function fields of genus 3 and 4. Low computational constructing complexity is an important constrain. The number of rational points has great interest for us.

The focus Nonhyperelliptic function fields as cubic fields with given discriminant

The background Kurt Hensel (1861-1941) was born in Königsberg (now Kaliningrad). He was grandnephew of Felix Mendelsohn-Bartholdy. His family moved to Berlin.. Hensel studies were in Berlin and Bonn. Kronecker supervised his doctoral studies at the University of Berlin. His approach to the theory of algebraic function fields was arithmetical [He-La].

The idea We construct nonhyperelliptic function fields over an algebraic closed field of characteristic 0 and reduce mod p

The reduction theory bases on work of Max Deuring (1907-1984). Emmy Noether supervised his doctoral studies at the University of Göttingen [De].



Kurt Hensel

The realization for genus three Let **F** be a nonhyperelliptic algebraic function field of genus 3 over an algebraic closed field **k** of characteristic zero. Hence there exists a function z allowing the construction of F as degree 3 extension **[F:k(z)].**
The proof uses the Riemann-Roch Theorem as substantial ingredient.

$$y^3+a(x)y+b(x)\in k[x,y]$$

$$a(x):=x^2+a_1x+a_2\in k[x] \qquad b(x):=x^4+b_1x^3+b_2x^2+b_3x+b_4\in k[x]$$

$$\Delta:=4a(x)^3+27b(x)^2\in k[x]$$

Some remarks

The genus 3 function fields of Picard curves:

$p>3,\ a(x)=0,\ b(x)\ separable$

A bad reduction case:

$p:=5,\ a(x):=x^2+x+1,\ b(x):=x^4+x^3+x^2+x+1,\ g=1$

Constructing a function field F of genus 4 over a finite field with q elements:

- q = 33 554 467
- Serre Bound: 33 600 808
- Oesterl'e Bound: 33 566 053
- N(F)= 33 561 509

$$a(x)=x^4+4870727x^3+15220770x^2+16867445x+18950748$$

$$b(x)=33554466x^6+24128110x^5+8860585x^4+16212979x^3+25450461x^2+11641003x$$

The realization for genus four Let F be a nonhyperelliptic algebraic function field of genus 4. There exist two types:

- Containing one divisor class of degree 3
- Containing two divisor classes of degree 3

Once again the ground of the proof is the Riemann-Roch Theorem

The first type:

$$y^3+a(x)y+b(x)\in k[x,y]$$

$$a(x):=x^4+a_1x^3+a_2x^2+a_3x+a_4\in k(x) \qquad b(x):=x^6+b_1x^5+b_2x^4+b_3x^3+b_4x^2+b_5x+b_6\in k[x]$$

$$\Delta:=4a(x)^3+27b(x)^2\in k[x]$$

The second type:

$$y^3+a(x)y^2+b(x)y+c(x)\in k[x]$$

$$a(x),b(x),c(x)\in k[x] \qquad deg\,a(x)=deg\,b(x)=deg\,c(x)=3$$

Some references :

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[He-La] K.Hensel, G. Landsberg. *Theorie der algebraischen Funktionen einer Variabeln.* B.G.Teubner, Leipzig, 1902.
[Ho-La] E.W. Howe, K.E. Lauter. Improved bounds for the number of points on curves over finite fields, *Annales d l'institut Fourier,* **53** no. 6 (2003), p. 1677-1737
[La-Se] K. Lauter, Jean-Pierre Serre. The maximum or minimum number of rational points on curves of genus three over finite fields, *Compositio Mathematica,* **134** (2002) p. 87-111.
[Ko-We]K. Koike, A.Weng. Construction of CM Picard Curves. Mathematics of Computation,**74**,249 :499-518,2004.